The following is a review of the Analysis of Fixed Income Investments principles designed to address the learning outcome statements set forth by CFA Institute®. This topic is also covered in:

Introduction to the Measurement of Interest Rate Risk

Study Session 16

Exam Focus

This topic review is about the relation of yield changes and bond price changes, primarily based on the concepts of duration and convexity. There is really nothing in this study session that can be safely ignored; the calculation of duration, the use of duration, and the limitations of duration as a measure of bond price risk are all important. You should work to understand what convexity is and its relation to the interest rate risk of fixed-income securities. There are two important formulas: the formula for effective duration and the formula for estimating the price effect of a yield change based on both duration and convexity. Finally, you should get comfortable with how and why the convexity of a bond is affected by the presence of embedded options.

LOS 67.a: Distinguish between the full valuation approach (the scenario analysis approach) and the duration/convexity approach for measuring interest rate risk, and explain the advantage of using the full valuation approach.

The full valuation or **scenario analysis approach** to measuring interest rate risk is based on applying the valuation techniques we have learned for a given change in the yield curve (i.e., for a given *interest rate scenario*). For a single option-free bond, this could be simply, "if the YTM increases by 50 bp or 100 bp, what is the impact on the value of the bond?" More complicated scenarios can be used as well, such as the effect on the bond value of a steepening of the yield curve (long-term rates increase more than short-term rates). If our valuation model is good, the exercise is straightforward: plug in the rates described in the interest rate scenario(s), and see what happens to the values of the bonds. For more complex bonds, such as callable bonds, a pricing model that incorporates yield volatility as well as specific yield curve change scenarios is required to use the full valuation approach. If the valuation models used are sufficiently good, this is the theoretically preferred approach. Applied to a portfolio of bonds, one bond at a time, we can get a very good idea of how different interest rate change scenarios will affect the value of the portfolio. Using this approach with extreme changes in interest rates is called **stress testing** a bond portfolio.

The **duration/convexity approach** provides an approximation of the actual interest rate sensitivity of a bond or bond portfolio. Its main advantage is its simplicity compared to the full valuation approach. The full valuation approach can get quite complex and time consuming for a portfolio of more than a few bonds, especially if some of the bonds have more complex structures, such as call provisions. As we will see shortly, limiting our scenarios to parallel yield curve shifts and "settling" for an estimate of interest rate risk allows us to use the summary measures, duration, and convexity. This greatly simplifies the process of estimating the value impact of overall changes in yield.

Compared to the duration/convexity approach, the full valuation approach is more precise and can be used to evaluate the price effects of more complex interest rate scenarios. Strictly speaking, the duration-convexity approach is appropriate only for estimating the effects of parallel yield curve shifts.

Example: The full valuation approach

Consider two option-free bonds. Bond X is an 8% annual-pay bond with five years to maturity, priced at 108.4247 to yield 6% (N = 5; PMT = 8.00; FV = 100; I/Y = 6.00%; CPT → PV = –108.4247).

Bond Y is a 5% annual-pay bond with 15 years to maturity, priced at 81.7842 to yield 7%.

Assume a $10 million face-value position in each bond and two scenarios. The first scenario is a parallel shift in the yield curve of +50 basis points and the second scenario is a parallel shift of +100 basis points. Note that the bond price of 108.4247 is the price per $100 of par value. With $10 million of par value bonds, the market value will be $10.84247 million.

Answer:

The full valuation approach for the two simple scenarios is illustrated in the following figure.

The Full Valuation Approach

		Market Value of:			
Scenario	*Yield Δ*	*Bond X (in millions)*	*Bond Y (in millions)*	*Portfolio*	*Portfolio Value Δ%*
Current	+0 bp	$10.84247	$8.17842	$19.02089	
1	+50 bp	$10.62335	$7.79322	$18.41657	–3.18%
2	+100 bp	$10.41002	$7.43216	$17.84218	–6.20%

N = 5; PMT = 8; FV = 100; I/Y = 6% + 0.5%; CPT → PV = –106.2335

N = 5; PMT = 8; FV = 100; I/Y = 6% + 1%; CPT → PV = –104.1002

N = 15; PMT = 5; FV = 100; I/Y = 7% + 0.5%; CPT → PV = –77.9322

N = 15; PMT = 5; FV = 100; I/Y = 7% + 1%; CPT → PV = –74.3216

Portfolio value change 50 bp: (18.41657 – 19.02089) / 19.02089 = –0.03177 = –3.18%

Portfolio value change 100 bp: (17.84218 – 19.02089) / 19.02089 = –0.06197 = –6.20%

It's worth noting that, on an individual bond basis, the effect of an increase in yield on the bonds' values is less for Bond X than for Bond Y (i.e., with a 50 bp increase in yields, the value of Bond X falls by 2.02%, while the value of Bond Y falls by 4.71%; and with a 100 bp increase, X falls by 3.99%, while Y drops by 9.12%). This, of course, is totally predictable since Bond Y is a longer-term bond and has a lower coupon—both of which mean more interest rate risk.

Professor's Note: Let's review the effects of bond characteristics on duration (price sensitivity). Holding other characteristics the same, we can state the following:

- *Higher (lower) coupon means lower (higher) duration.*
- *Longer (shorter) maturity means higher (lower) duration.*
- *Higher (lower) market yield means lower (higher) duration.*

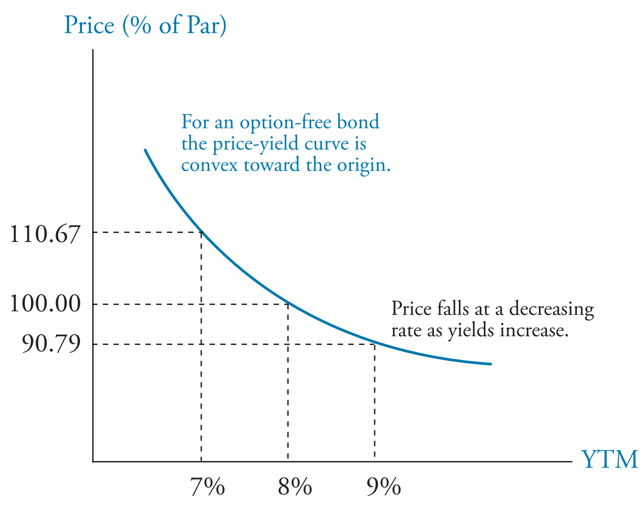
Finance professors love to test these relations.

LOS 67.b: Demonstrate the price volatility characteristics for option-free, callable, prepayable, and putable bonds when interest rates change.

LOS 67.c: Describe positive convexity, negative convexity, and their relation to bond price and yield.

We established earlier that the relation between price and yield for a straight coupon bond is negative. An increase in yield (discount rate) leads to a decrease in the value of a bond. The precise nature of this relationship for an option-free, 8%, 20-year bond is illustrated in Figure 1.

Figure 1: Price-Yield Curve for an Option-Free, 8%, 20-Year Bond



First, note that the price-yield relationship is negatively sloped, so the price falls as the yield rises. Second, note that the relation follows a curve, not a straight line. Since the curve is convex (toward the origin), we say that an option-free bond has **positive convexity**. Because of its positive convexity, the price of an option-free bond *increases more when yields fall than it decreases when yields rise*. In Figure 1 we have illustrated that, for an 8%, 20-year option-free bond, a 1% decrease in the YTM will increase the price to 110.67, a *10.67% increase* in price. A 1% increase in YTM will cause the bond value to decrease to 90.79, a *9.22% decrease* in value.

If the price-yield relation were a straight line, there would be no difference between the price increase and the price decline in response to equal decreases and increases in yields. Convexity is a good thing for a bond owner; for a given volatility of yields, price increases are larger than price decreases. The convexity property is often expressed by saying, "a bond's price falls at a decreasing rate as yields rise." For the price-yield relationship to be convex, the slope (rate of decrease) of the curve must be decreasing as we move from left to right (i.e., towards higher yields).

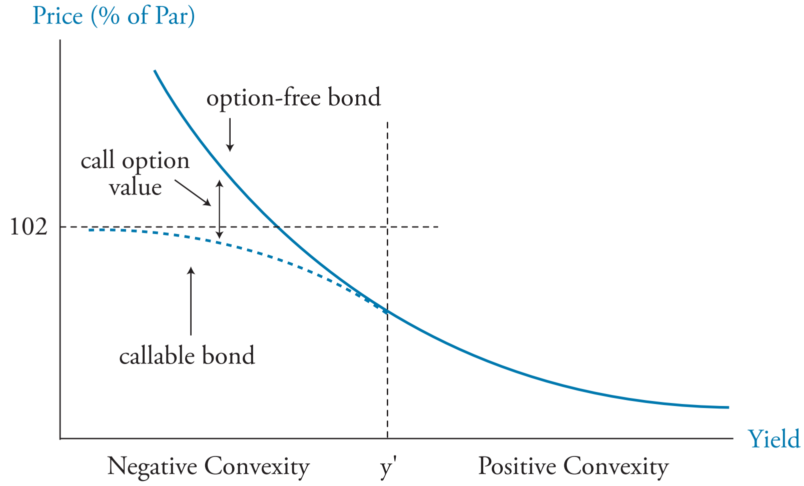
Note that the duration (interest rate sensitivity) of a bond at any yield is (absolute value of) the slope of the price-yield function at that yield. The convexity of the price-yield relation for an option-free bond can help you remember a result presented earlier, that the duration of a bond is less at higher market yields.

Callable Bonds, Prepayable Securities, and Negative Convexity

With a **callable** or **prepayable debt**, the upside price appreciation in response to decreasing yields is limited (sometimes called price compression). Consider the case of a bond that is currently callable at 102. The fact that the issuer can call the bond at any time for $1,020 per $1,000 of face value puts an effective upper limit on the value of the bond. As Figure 2 illustrates, as yields fall and the price approaches $1,020, the price-yield curve rises more slowly than that of an identical but noncallable bond. When the price begins to *rise at a decreasing rate* in response to further decreases in yield, the price-yield curve "bends over" to the left and exhibits **negative convexity**.

Thus, in Figure 2, so long as yields remain *below level y',* callable bonds will exhibit *negative convexity;* however, at yields *above level y',* those same callable bonds will exhibit *positive convexity.* In other words, at higher yields the value of the call options becomes very small so that a callable bond will act very much like a noncallable bond. It is only at lower yields that the callable bond will exhibit negative convexity.

Figure 2: Price-Yield Function of a Callable vs. an Option-Free Bond



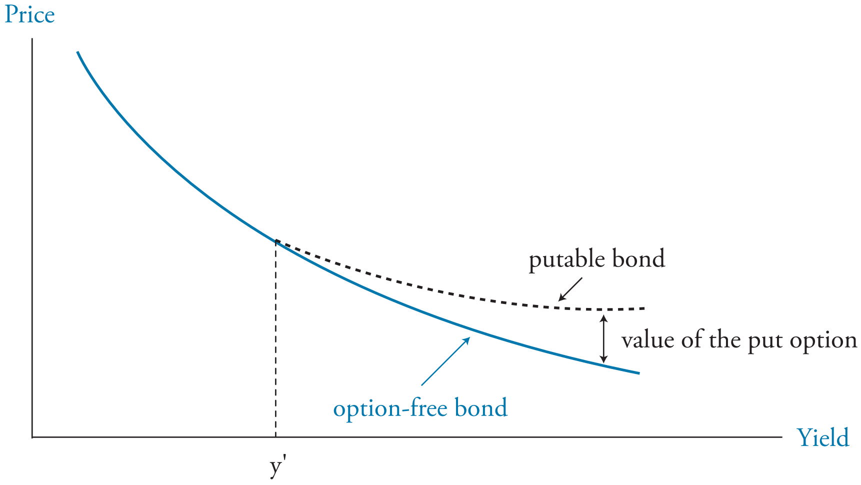
In terms of price sensitivity to interest rate changes, the slope of the price-yield curve at any particular yield tells the story. Note that as yields fall, the slope of the price-yield curve for the callable bond decreases, becoming almost zero (flat) at very low yields. This tells us how a call feature affects price sensitivity to changes in yield. At higher yields, the interest rate risk of a callable bond is very close or identical to that of a similar option-free bond. At lower yields, the price volatility of the callable bond will be much lower than that of an identical but noncallable bond.

The effect of a prepayment option is quite similar to that of a call; at low yields it will lead to negative convexity and reduce the price volatility (interest rate risk) of the security. Note that when yields are low and callable and prepayable securities exhibit less interest rate risk, reinvestment risk rises. At lower yields, the probability of a call and the prepayment rate both rise, increasing the risk of having to reinvest principal repayments at the lower rates.

The Price Volatility Characteristics of Putable Bonds

The value of a put increases at higher yields and decreases at lower yields opposite to the value of a call option. Compared to an option-free bond, a **putable bond** will have *less* price volatility at higher yields. This comparison is illustrated in Figure 3.

Figure 3: Comparing the Price-Yield Curves for Option-Free and Putable Bonds



In Figure 3, the price of the putable bond falls more slowly in response to increases in yield above y' because the value of the embedded put rises at higher yields. The slope of the price-yield relation is flatter, indicating less price sensitivity to yield changes (lower duration) for the putable bond at higher yields. At yields below y', the value of the put is quite small, and a putable bond's price acts like that of an option-free bond in response to yield changes.

LOS 67.d: Calculate and interpret the effective duration of a bond, given information about how the bond's price will increase and decrease for given changes in interest rates.

In our introduction to the concept of duration, we described it as the ratio of the percentage change in price to change in yield. Now that we understand convexity, we know that the price change in response to rising rates is smaller than the price change in response to falling rates for option-free bonds. The formula we will use for calculating the **effective duration** of a bond uses the average of the price changes in response to equal increases and decreases in yield to account for this fact. If we have a callable bond that is trading in the area of negative convexity, the price increase is smaller than the price decrease, but using the average still makes sense.

The formula for calculating the effective duration of a bond is:

$$\text{effective duration} = \frac{(\text{bond price when yields fall} - \text{bond price when yields rise})}{2 \times (\text{initial price}) \times (\text{change in yield in decimal form})}$$

which we will sometimes write as $\text{duration} = \frac{V_- - V_+}{2V_0(\Delta y)}$

where:
V_- = bond value if the yield decreases by Δy
V_+ = bond value if the yield increases by Δy
V_0 = initial bond price
Δy = change in yield used to get V_- and V_+, *expressed in decimal form*

Consider the following example of this calculation.

Example: Calculating effective duration

Consider a 20-year, semiannual-pay bond with an 8% coupon that is currently priced at $908.00 to yield 9%. If the yield declines by 50 basis points (to 8.5%), the price will increase to $952.30, and if the yield increases by 50 basis points (to 9.5%), the price will decline to $866.80. Based on these price and yield changes, calculate the effective duration of this bond.

Answer:

Let's approach this intuitively to gain a better understanding of the formula. We begin by computing the average of the percentage change in the bond's price for the yield increase and the percentage change in price for a yield decrease. We can calculate this as:

$$\text{average } \textit{percentage} \text{ price change} = \frac{(\$952.30 - \$866.80)}{2 \times \$908.00} = 0.0471\%, \text{ or } 4.71\%$$

The 2 in the denominator is to obtain the average price change, and the $908 in the denominator is to obtain this average change as a percentage of the current price.

To get the duration (to scale our result for a 1% change in yield), the final step is to divide this average percentage price change by the change in interest rates that caused it. In the example, the yield change was 0.5%, which we need to write in decimal form as 0.005. Our estimate of the duration is:

$$\frac{0.0471}{0.005} = \frac{4.71\%}{0.50\%} = 9.42 = \text{duration}$$

Using the formula previously given, we have:

$$\text{effective duration} = \frac{(\$952.3 - \$866.8)}{2 \times \$908 \times 0.005} = 9.416$$

The interpretation of this result, as you should be convinced by now, is that a 1% change in yield produces an approximate change in the price of this bond of 9.42%. Note, however, that this estimate of duration was based on a change in yield of 0.5% and will perform best for yield changes close to this magnitude. Had we used a yield change of 0.25% or 1%, we would have obtained a slightly different estimate of effective duration.

This is an important concept, and you are required to learn the formula for the calculation. To further help you understand this formula and remember it, consider the following.

The price increase in response to a 0.5% decrease in rates was $\frac{\$44.30}{\$908} = 4.879\%$.

The price decrease in response to a 0.5% increase in rates was $\frac{\$41.20}{\$908} = 4.537\%$.

The average of the percentage price increase and the percentage price decrease is 4.71%. Since we used a 0.5% change in yield to get the price changes, we need to double this and get a 9.42% change in price for a 1% change in yield. The duration is 9.42.

For bonds with no embedded options, modified duration and effective duration will be equal or very nearly equal. In order to calculate effective duration for a bond with an embedded option, we need a pricing model that takes account of how the cash flows change when interest rates change.

LOS 67.e: Calculate the approximate percentage price change for a bond, given the bond's effective duration and a specified change in yield.

Multiply effective duration by the change in yield to get the magnitude of the price change and then change the sign to get the direction of the price change right (yield up, price down).

percentage change in bond price = –effective duration × change in yield in percent

Example: Using effective duration

What is the expected percentage price change for a bond with an effective duration of nine in response to an increase in yield of 30 basis points?

Answer:

$-9 \times 0.3\% = -2.7\%$

We expect the bond's price to decrease by 2.7% in response to the yield change. If the bond were priced at $980, the new price is 980 × (1 – 0.027) = $953.54.

LOS 67.f: Distinguish among the alternative definitions of duration and explain why effective duration is the most appropriate measure of interest rate risk for bonds with embedded options.

The formula we used to calculate duration based on price changes in response to equal increases and decreases in YTM, $\text{duration} = \frac{V_- - V_+}{2V_0(\Delta y)}$, is the formula for effective (option-adjusted) duration. This is the preferred measure because it gives a good approximation of interest rate sensitivity for both option-free bonds and *bonds with embedded options*.

Macaulay duration is an estimate of a bond's interest rate sensitivity based on the time, in years, until promised cash flows will arrive. Since a 5-year zero-coupon bond has only one cash flow five years from today, its Macaulay duration is five. The change in value in response to a 1% change in yield for a 5-year zero-coupon bond is approximately 5%. A 5-year coupon bond has some cash flows that arrive earlier than five years from today (the coupons), so its Macaulay duration is less than five. This is consistent with what we learned earlier: the higher the coupon, the less the price sensitivity (duration) of a bond.

Macaulay duration is the earliest measure of duration, and because it was based on the time, duration is often stated as years. Because Macaulay duration is based on the expected cash flows for an option-free bond, it is not an appropriate estimate of the price sensitivity of bonds with embedded options.

Modified duration is derived from Macaulay duration and offers a slight improvement over Macaulay duration in that it takes the current YTM into account. Like Macaulay duration, and for the same reasons, modified duration is not an appropriate measure of interest rate sensitivity for bonds with embedded options. For option-free bonds, however, effective duration (based on small changes in YTM) and modified duration will be very similar.

Professor's Note: The LOS here do not require that you calculate either Macaulay duration or modified duration, only effective duration. For your own understanding, however, note that the relation is

$$\textit{modified duration} = \frac{\textit{Macaulay duration}}{1 + \textit{periodic market yield}}$$

. This accounts for the fact we learned earlier that duration decreases as YTM increases. Graphically, the slope of the price-yield curve is less steep at higher yields.

Effective Duration for Bonds With Embedded Options

As noted earlier, in comparing the various duration measures, both Macaulay and modified duration are calculated directly from the promised cash flows for a bond with no adjustment for the effect of any embedded options on cash flows. Effective duration is calculated from expected price changes in response to changes in yield that explicitly take into account a bond's option provisions (i.e., they are in the price-yield function used).

Interpreting Duration

We can interpret duration in three different ways.

First, duration is the slope of the price-yield curve at the bond's current YTM. Mathematically, the slope of the price-yield curve is the first derivative of the price-yield curve with respect to yield.

A second interpretation of duration, as originally developed by Macaulay, is a weighted average of the time (in years) until each cash flow will be received. The weights are the proportions of the total bond value that each cash flow represents. The answer, again, comes in years.

A third interpretation of duration is the approximate percentage change in price for a 1% change in yield. This interpretation, price sensitivity in response to a change in yield, is the preferred, and most intuitive, interpretation of duration.

Professor's Note: The fact that duration was originally calculated and expressed in years has been a source of confusion for many candidates and finance students. Practitioners regularly speak of "longer duration securities." This confusion is the reason for this part of the LOS. The most straightforward interpretation of duration is the one that we have used up to this point: "It is the approximate percentage change in a bond's price for a 1% change in YTM." I have seen duration expressed in years in CFA exam questions; just ignore the years and use the number. I have also seen questions asking whether duration becomes longer or shorter in response to a change; longer means higher or more interest rate sensitivity. A duration of 6.82 years means that for a 1% change in YTM, a bond's value will change approximately 6.82%. This is the best way to "interpret" duration.

LOS 67.g: Calculate the duration of a portfolio, given the duration of the bonds comprising the portfolio, and explain the limitations of portfolio duration.

The concept of duration can also be applied to portfolios. In fact, one of the benefits of duration as a measure of interest rate risk is that the **duration of a portfolio** is simply the weighted average of the durations of the individual securities in the portfolio. Mathematically, the duration of a portfolio is:

$$\text{portfolio duration} = w_1D_1 + w_2D_2 + \ldots + w_ND_N$$

where:
w_i = market value of bond i divided by the market value of the portfolio
D_i = the duration of bond i
N = the number of bonds in the portfolio

Example: Calculating portfolio duration

Suppose you have a two-security portfolio containing Bonds A and B. The market value of Bond A is $6,000, and the market value of Bond B is $4,000. The duration of Bond A is 8.5, and the duration of Bond B is 4.0. Calculate the duration of the portfolio.

Answer:

First, find the weights of each bond. Since the market value of the portfolio is $10,000 = $6,000 + $4,000, the weight of each security is as follows:

$$\text{weight in Bond A} = \frac{\$6{,}000}{\$10{,}000} = 60\%$$

$$\text{weight in Bond B} = \frac{\$4{,}000}{\$10{,}000} = 40\%$$

Using the formula for the duration of a portfolio, we get:

$$\text{portfolio duration} = (0.6 \times 8.5) + (0.4 \times 4.0) = 6.7$$

Limitations of Portfolio Duration

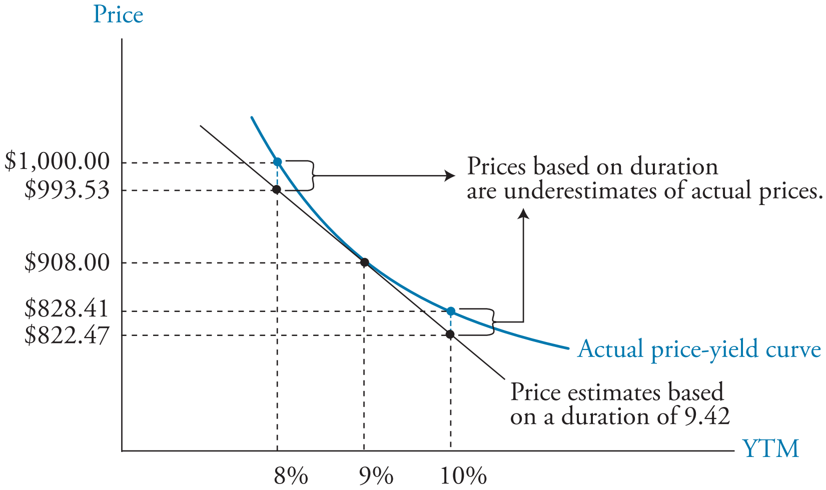
The limitations of portfolio duration as a measure of interest rate sensitivity stem from the fact that yields may not change equally on all the bonds in the portfolio. With a portfolio that includes bonds with different maturities, credit risks, and embedded options, there is no reason to suspect that the yields on individual bonds will change by equal amounts when the yield curve changes. As an example, a steepening of the yield curve can increase yields on long-term bonds and leave the yield on short-term bonds unchanged. It is for this reason that we say that duration is a good measure of the sensitivity of portfolio value to *parallel* changes in the yield curve.

LOS 67.h: Describe the convexity measure of a bond and estimate a bond's percentage price change, given the bond's duration and convexity and a specified change in interest rates.

Convexity is a measure of the curvature of the price-yield curve. The more curved the price-yield relation is, the greater the convexity. A straight line has a convexity of zero. If the price-yield "curve" were, in fact, a straight line, the convexity would be zero. The reason we care about convexity is that the more curved the price-yield relation is, the worse our duration-based estimates of bond price changes in response to changes in yield are.

As an example, consider again an 8%, 20-year Treasury bond priced at $908 so that it has a yield to maturity of 9%. We previously calculated the effective duration of this bond as 9.42. Figure 4 illustrates the differences between actual bond price changes and duration-based estimates of price changes at different yield levels.

Figure 4: Duration-Based Price Estimates vs. Actual Bond Prices



Based on a value of 9.42 for duration, we would estimate the new prices after 1% changes in yield (to 8% and to 10%) as 1.0942 × 908 = $993.53 and (1 – 0.0942) × 908 = $822.47, respectively. These price estimates are shown in Figure 4 along the straight line tangent to the actual price-yield curve.

The actual price of the 8% bond at a YTM of 8% is, of course, par value ($1,000). Based on a YTM of 10%, the actual price of the bond is $828.41, about $6 higher than our duration based estimate of $822.47. Note that price estimates based on duration are less than the actual prices for both a 1% increase and a 1% decrease in yield.

Figure 4 illustrates why convexity is important and why estimates of price changes based solely on duration are inaccurate. If the price-yield relation were a straight line (i.e., if convexity were zero), duration alone would provide good estimates of bond price changes for changes in yield of any magnitude. The greater the convexity, the greater the error in price estimates based solely on duration.

A Bond's Approximate Percentage Price Change Based on Duration and Convexity

By combining duration and convexity, we can obtain a more accurate estimate of the percentage change in price of a bond, especially for relatively large changes in yield. The formula for estimating a bond's percentage price change based on its convexity and duration is:

$$\text{percentage change in price} = \text{duration effect} + \text{convexity effect}$$

$$= \left\{\left[-\text{duration} \times (\Delta y)\right] + \left[\text{convexity} \times (\Delta y)^2\right]\right\} \times 100$$

With Δy entered as a decimal, the "× 100" is necessary to get an answer in percent.

Example: Estimating price changes with duration and convexity

Consider an 8% Treasury bond with a current price of $908 and a YTM of 9%. Calculate the percentage change in price of both a 1% increase and a 1% decrease in YTM based on a duration of 9.42 and a convexity of 68.33.

Answer:

The duration effect, as we calculated earlier, is 9.42 × 0.01 = 0.0942 = 9.42%. The convexity effect is $68.33 \times 0.01^2 \times 100 = 0.00683 \times 100 = 0.683\%$. The total effect for a *decrease in yield of 1%* (from 9% to 8%) is 9.42% + 0.683% = +10.103%, and the estimate of the new price of the bond is 1.10103 × 908 = 999.74. This is much closer to the actual price of $1,000 than our estimate using only duration.

The total effect for an *increase in yield of 1%* (from 9% to 10%) is –9.42% + 0.683% = –8.737%, and the estimate of the bond price is (1 – 0.08737)(908) = $828.67. Again, this is much closer to the actual price ($828.40) than the estimate based solely on duration.

There are a few points worth noting here. First, the convexity adjustment is always positive when convexity is positive because $(\Delta y)^2$ is always positive. This goes along with the illustration in Figure 4, which shows that the duration-only based estimate of a bond's price change suffered from being an underestimate of the percentage increase in the bond price when yields fell, and an overestimate of the percentage decrease in the bond price when yields rose. Recall, that for a callable bond, convexity can be negative at low yields. When convexity is negative, the convexity adjustment to the duration-only based estimate of the percentage price change will be negative for both yield increases and yield decreases.

Professor's Note: Different dealers may calculate the convexity measure differently. Often the measure is calculated in a way that requires us to divide the measure by two in order to get the correct convexity adjustment. For exam purposes, the formula we've shown here is the one you need to know. However, you should also know that there can be some variation in how different dealers calculate convexity.

LOS 67.i: Differentiate between modified convexity and effective convexity.

Effective convexity takes into account changes in cash flows due to embedded options, while modified convexity does not. The difference between modified convexity and effective convexity mirrors the difference between modified duration and effective duration. Recall that modified duration is calculated without any adjustment to a bond's cash flows for embedded options. Also recall that effective duration was appropriate for bonds with embedded options because the inputs (prices) were calculated under the assumption that the cash flows could vary at different yields because of the embedded options in the securities. Clearly, effective convexity is the appropriate measure to use for bonds with embedded options, since it is based on bond values that incorporate the effect of embedded options on the bond's cash flows.

LOS 67.j: Calculate the price value of a basis point (PVBP), and explain its relationship to duration.

The **price value of a basis point** (PVBP) is the dollar change in the price/value of a bond or a portfolio when the yield changes by one basis point, or 0.01%. We can calculate the PVBP directly for a bond by changing the YTM by one basis point and computing the change in value. As a practical matter, we can use duration to calculate the price value of a basis point as:

price value of a basis point = duration × 0.0001 × bond value

The following example demonstrates this calculation.

Example: Calculating the price value of a basis point

A bond has a market value of $100,000 and a duration of 9.42. What is the price value of a basis point?

Answer:

Using the duration formula, the percentage change in the bond's price for a change in yield of 0.01% is 0.01% × 9.42 = 0.0942%. We can calculate 0.0942% of the original $100,000 portfolio value as 0.000942 × 100,000 = $94.20. If the bond's yield increases (decreases) by one basis point, the portfolio value will fall (rise) by $94.20. $94.20 is the (duration-based) price value of a basis point for this bond.

We could also directly calculate the price value of a basis point for this bond by increasing the YTM by 0.01% (0.0001) and calculating the change in bond value. This would give us the PVBP for an increase in yield. This would be very close to our duration-based estimate because duration is a very good estimate of interest rate risk for small changes in yield. We can ignore the convexity adjustment here because it is of very small magnitude: $(\Delta y)^2 = (0.0001)^2 = 0.00000001$, which is very small indeed!

LOS 67.k: Discuss the impact of yield volatility on the interest rate risk of a bond.

Earlier in this topic review, we introduced duration as a measure of interest rate risk. A bond with a lower duration is less affected by a given change in yield than a bond with greater duration. Here we combine a bond's duration with its yield volatility in assessing its interest rate risk.

Consider a Treasury bond with a duration of 7 and a similar single-B rated corporate bond with a duration of 5. Based on duration alone, we would say that the Treasury bond has more interest rate risk. If, however, the volatility of the market yield on the corporate bond is sufficiently greater than the volatility of the market yield on the Treasury bond, the corporate bond can have greater price volatility due to yield (interest rate) changes than the Treasury bond. Investors must consider both the effects of yield changes on bond values and how volatile yields are when estimating interest rate risk. One measure of price risk that considers both these components is value-at-risk (VaR).

Key Concepts

LOS 67.a

The full valuation approach to measuring interest rate risk involves using a pricing model to value individual bonds and can be used to find the price impact of any scenario of interest rate/yield curve changes. Its advantages are its flexibility and precision.

The duration/convexity approach is based on summary measures of interest rate risk and, while simpler to use for a portfolio of bonds than the full valuation approach, is theoretically correct only for parallel shifts in the yield curve.

LOS 67.b

Callable bonds and prepayable securities will have less price volatility (lower duration) at low yields, compared to option-free bonds.

Putable bonds will have less price volatility at high yields, compared to option-free bonds.

LOS 67.c

Option-free bonds have a price-yield relationship that is curved (convex toward the origin) and are said to exhibit positive convexity. In this case, bond prices fall less in response to an increase in yield than they rise in response to an equal-sized decrease in yield.

Callable bonds exhibit negative convexity at low yield levels. In this case, bond prices rise less in response to a decrease in yield than they fall in response to an equal-sized increase in yield.

LOS 67.d

Effective duration is calculated as the ratio of the average percentage price change for an equal-sized increase and decrease in yield, to the change in yield.

$$\text{effective duration} = \frac{V_{-} - V_{+}}{2V_0(\Delta y)}$$

LOS 67.e

Approximate percentage change in bond price = –duration × change in yield in percent.

LOS 67.f

The most intuitive interpretation of duration is as the percentage change in a bond's price for a 1% change in yield to maturity.

Macaulay duration and modified duration are based on a bond's promised cash flows.

Effective duration is appropriate for estimating price changes in bonds with embedded options because it takes into account the effect of embedded options on a bond's cash flows.

LOS 67.g

The duration of a bond portfolio is equal to a weighted average of the individual bond durations, where the weights are the proportions of total portfolio value in each bond position.

Portfolio duration is limited because it gives the sensitivity of portfolio value only to yield changes that are equal for all bonds in the portfolio, an unlikely scenario for most portfolios.

LOS 67.h

Because of convexity, the duration measure is a poor approximation of price sensitivity for yield changes that are not absolutely small. The convexity adjustment accounts for the curvature of the price-yield relationship.

Incorporating both duration and convexity, we can estimate the percentage change in price in response to a change in yield of (Δy) as:

$$\left\{\left[(-\text{duration})(\Delta y)\right] + \left[(\text{convexity})(\Delta y)^2\right]\right\} \times 100$$

LOS 67.i

Effective convexity considers expected changes in cash flows that may occur for bonds with embedded options, while modified convexity does not.

LOS 67.j

Price value of a basis point (PVBP) is an estimate of the change in a bond's or a bond portfolio's value for a one basis point change in yield.

$$\text{PVBP} = \text{duration} \times 0.0001 \times \text{bond (or portfolio) value}$$

LOS 67.k

Yield volatility is the standard deviation of the changes in the yield of a bond. Uncertainty about a bond's future price due to changes in yield results from both a bond's price sensitivity to yield changes (its duration) and also from the volatility of its yield in the market.

CONCEPT CHECKERS

1. Why is the price/yield profile of a callable bond less convex than that of an otherwise identical option-free bond? The price:
   A. increase is capped from above, at or near the call price as the required yield decreases.
   B. increase is capped from above, at or near the call price as the required yield increases.
   C. decrease is limited from below, at or near the call price as the required yield increases.

2. The 4.65% semiannual-pay Portage Health Authority bonds have exactly 17 years to maturity and are currently priced to yield 4.39%. Using the full valuation approach, the interest rate exposure (in percent of value) for these bonds, given a 75 basis point increase in required yield, is *closest* to:
   A. –9.104%.
   B. –9.031%.
   C. –8.344%.

3. A 14% semiannual-pay coupon bond has six years to maturity. The bond is currently trading at par. Using a 25 basis point change in yield, the effective duration of the bond is *closest* to:
   A. 0.389.
   B. 3.889.
   C. 3.970.

4. Suppose that the bond in Question 3 is callable at par today. Using a 25 basis point change in yield, the bond's effective duration assuming that its price cannot exceed 100 is *closest* to:
   A. 1.972.
   B. 1.998.
   C. 19.72.

5. The modified duration of a bond is 7.87. The percentage change in price using duration for a yield decrease of 110 basis points is *closest* to:
   A. –8.657%.
   B. +7.155%.
   C. +8.657%.

6. A bond has a convexity of 57.3. The convexity effect if the yield decreases by 110 basis points is *closest* to:
   A. –1.673%.
   B. +0.693%.
   C. +1.673%.

7. Assume a bond has an effective duration of 10.5 and a convexity of 97.3. Using both of these measures, the estimated percentage change in price for this bond, in response to a decline in yield of 200 basis points, is *closest* to:
   A. 19.05%.
   B. 22.95%.
   C. 24.89%.

8. An analyst has determined that if market yields rise by 100 basis points, a certain high-grade corporate bond will have a convexity effect of 1.75%. Further, she's found that the total estimated percentage change in price for this bond should be –13.35%. Given this information, it follows that the bond's percentage change in price due to duration is:
   A. –15.10%.
   B. –11.60%.
   C. +16.85%.

9. The total price volatility of a typical noncallable bond can be found by:
   A. adding the bond's convexity effect to its effective duration.
   B. adding the bond's negative convexity to its modified duration.
   C. subtracting the bond's negative convexity from its positive convexity.

10. The current price of a $1,000, 7-year, 5.5% semiannual coupon bond is $1,029.23. The bond's PVBP is *closest* to:
   A. $0.05.
   B. $0.60.
   C. $5.74.

11. The effect on a bond portfolio's value of a decrease in yield would be *most accurately* estimated by using:
   A. the full valuation approach.
   B. the price value of a basis point.
   C. both the portfolio's duration and convexity.

12. An analyst has noticed lately that the price of a particular bond has risen less when the yield falls by 0.1% than the price falls when rates increase by 0.1%. She could conclude that the bond:
   A. is an option-free bond.
   B. has an embedded put option.
   C. has negative convexity.

13. Which of the following measures is *lowest* for a currently callable bond?
   A. Macaulay duration.
   B. Effective duration.
   C. Modified duration.

COMPREHENSIVE PROBLEMS

Use the following information to answer Questions 1 through 6.

A bond dealer provides the following selected information on a portfolio of fixed-income securities.

Par Value	*Mkt. Price*	*Coupon*	*Modified Duration*	*Effective Duration*	*Effective Convexity*
$2 million	100	6.5%	8	8	154
$3 million	93	5.5%	6	1	50
$1 million	95	7%	8.5	8.5	130
$4 million	103	8%	9	5	–70

1. What is the effective duration for the portfolio?
2. Calculate the price value of a basis point for this portfolio.
3. Which bond(s) likely has (have) no embedded options? (identify bonds by coupon)
4. Which bond(s) is (are) likely callable?
5. Which bond(s) is (are) likely putable?
6. What is the approximate price change for the 7% bond if its yield to maturity increases by 25 basis points?
7. Why might two bond dealers differ in their estimates of a portfolio's effective duration?
8. Why might portfolio effective duration be an inadequate measure of interest rate risk for a bond portfolio even if we assume the bond effective durations are correct?

Answers – Concept Checkers

1. **A** As the required yield decreases on a callable bond, the rate of increase in the price of the bond begins to slow down and eventually level off as it approaches the call price, a characteristic known as "negative convexity."

2. **C** We need to compare the value of the bond today to the value if the YTM increases by 0.75%.

   Price today = 103.092

   $$N = 34;\ PMT = \frac{4.65}{2} = 2.325;\ FV = 100;$$

   $$I/Y = \frac{4.39}{2} = 2.195\%;\ CPT \rightarrow PV = -103.092$$

   Price after a 75 basis point increase in the YTM is 94.490

   $$N = 34;\ PMT = \frac{4.65}{2} = 2.325;\ FV = 100;$$

   $$I/Y = \frac{5.14}{2} = 2.57\%;\ CPT \rightarrow PV = -94.490$$

   $$\text{Interest rate exposure} = \frac{94.490 - 103.092}{103.092} = -8.344\%$$

3. **C** $V_- = 100.999$

   $$N = 12;\ PMT = \frac{14.00}{2} = 7.00;\ FV = 100;$$

   $$I/Y = \frac{13.75}{2} = 6.875\%;\ CPT \rightarrow PV = -100.999$$

   $V_+ = 99.014$

   $$N = 12;\ PMT = \frac{14.00}{2} = 7.00;\ FV = 100;$$

   $$I/Y = \frac{14.25}{2} = 7.125\%;\ CPT \rightarrow PV = -99.014$$

   $V_0 = 100.000$

   $\Delta y = 0.0025$

   $$\text{duration} = \frac{V_- - V_+}{2V_0(\Delta y)} = \frac{100.999 - 99.014}{2(100)0.0025} = 3.970$$

4. **A** $V_- = 100$

$V_+ = 99.014$

$V_0 = 100$

$\Delta y = 0.0025$

$$\text{duration} = \frac{V_- - V_+}{2V_0(\Delta y)} = \frac{100 - 99.014}{2(100)0.0025} = 1.972$$

5. **C** $\text{Est.}[\Delta V\_\%] = -7.87 \times (-1.10\%) = 8.657\%$

6. **B** convexity effect = convexity × $(\Delta y)^2 = \left[57.3(0.011)^2\right] \times 100 = 0.693\%$

7. **C** Total estimated price change = (duration effect + convexity effect)

$$\left\{\left[-10.5 \times (-0.02)\right] + \left[97.3 \times (-0.02)^2\right]\right\} \times 100 = 21.0\% + 3.89\% = 24.89\%$$

8. **A** Total percentage change in price = duration effect + convexity effect. Thus:

$-13.35 = \text{duration effect} + 1.75 \Rightarrow \text{duration effect} = -15.10\%\cdot$

(Note: the duration effect must be negative because yields are rising.)

9. **A** Total percentage change in price = duration effect + convexity effect. Thus:

Total percentage change in price = effective duration + convexity effect.

(Note: since this is a noncallable bond, you can use either effective or modified duration in the above equation.)

10. **B** PVBP = initial price – price if yield is changed by 1 bp. First, we need to calculate the yield so that we can calculate the price of the bond with a 1 basis point change in yield. Using a financial calculator: PV = –1,029.23; FV = 1,000; PMT = 27.5 = (0.055 × 1,000) / 2; N =14 = 2 × 7 years; CPT → I/Y = 2.49998, multiplied by 2 = 4.99995, or 5.00%. Next, compute the price of the bond at a yield of 5.00% + 0.01%, or 5.01%. Using the calculator: FV = 1,000; PMT = 27.5; N = 14; I/Y = 2.505 (5.01 / 2); CPT → PV = $1,028.63. Finally, PVBP = $1,029.23 – $1,028.63 = $0.60.

11. **A** The full valuation approach is the most complex method, but also the most accurate.

12. **C** A bond with negative convexity will rise less in price in response to a decrease in yield than it will fall in response to an equal-sized increase in rates.

13. **B** The interest rate sensitivity of a bond with an embedded call option will be less than that of an option-free bond. Effective duration takes the effect of the call option into account and will, therefore, be less than Macaulay or modified duration.

Answers – Comprehensive Problems

1. Portfolio effective duration is the weighted average of the effective durations of the portfolio bonds.

   Numerators in weights are market values (par value × price as percent of par). Denominator is total market value of the portfolio.

   $$\frac{2}{9.86}(8)+\frac{2.79}{9.86}(1)+\frac{0.95}{9.86}(8.5)+\frac{4.12}{9.86}(5)=4.81 \text{ (weights are in millions)}$$

2. Price value of a basis point can be calculated using effective duration for the portfolio and the portfolio's market value, together with a yield change of 0.01%. Convexity can be ignored for such a small change in yield.

   4.81 × 0.0001 × 9,860,000 = $4,742.66

3. The 6.5% and 7% coupon bonds likely have no embedded options. For both of these bonds, modified duration and effective duration are identical, which would be the case if they had no embedded options. (It is possible that these bonds have options that are so far out of the money that the bond prices act as if there is no embedded option. One example might be a conversion option to common stock at $40 per share when the market value of the shares is $2.)

4. The 8% bond is likely callable. It is trading at a premium, its effective duration is less than modified duration, and it exhibits negative convexity. Remember, call price can be above par.

5. The 5.5% bond is likely putable. It is trading at a significant discount, its effective duration is much lower than its modified duration (close to zero in fact), and its convexity is positive but low. Note that a putable bond may trade below par when the put price is below par (also if there is risk that the issuer cannot honor the put). If it were callable, we would expect its modified and effective durations to be closer in value because the market price is significantly below likely call prices.

6. Based on the effective duration and effective convexity of the 7% bond, the approximate price change is:

   $[-8.5 \times 0.0025] + [130 \times 0.0025^2] \times 950{,}000 = -\$19{,}415.63$

7. In order to estimate effective duration, the dealers must use a pricing model for the bonds and choose a specific yield change. Differences in models or the yield change used can lead to differences in their estimates of effective duration.

8. Effective duration is based on small changes in yield and is appropriate for parallel changes in the yield curve (or equal changes in the yields to maturity for all portfolio bonds). Other types of yield changes will make portfolio duration an inadequate measure of portfolio interest rate risk.

SELF-TEST: FIXED INCOME INVESTMENTS

14 questions, 21 minutes

1. An estimate of the price change for an option-free bond caused by a 1% decline in its yield to maturity based only on its modified duration will result in an answer that:
   A. is too small.
   B. is too large.
   C. may be too small or too large.

2. Alfred LeBon purchased a semiannual pay, 7%, U.S. Treasury bond with 19 years to maturity for 91.16 the day after it had made a coupon payment. Two years later, the YTM was unchanged when he sold the bond. LeBon's gain when he sold the bond is *closest* to:
   A. 0.483%.
   B. 0.733%.
   C. 0.931%.

3. Which statement about the theories of the term structure of interest rates is *most accurate*?
   A. Under the liquidity preference theory, the yield curve will be positively sloped.
   B. A yield curve that slopes up and then down (humped) is consistent with the market segmentation theory but not with the pure expectations theory.
   C. Evidence that life insurance companies have a strong preference for 30-year bonds supports the market segmentation theory.

4. Which of the following is *least likely* a common form of external credit enhancement?
   A. Portfolio insurance.
   B. A corporate guarantee.
   C. A letter of credit from a bank.

5. A bond with an embedded put option has a modified duration of 7, an effective duration of 6 and a convexity of 62.5. If interest rates rise 25 basis points, the bond's price will change by *approximately*:
   A. 1.46%.
   B. 1.50%.
   C. 1.54%.

6. Which of the following bonds would be the best one to own if the yield curve shifts down by 50 basis points at all maturities?
   A. 4-year 8%, 8% YTM.
   B. 5-year 8%, 7.5% YTM.
   C. 5-year 8.5%, 8% YTM.

7. Which of the following provisions would *most likely* decrease the yield to maturity on a debt security?
   A. Call option.
   B. Conversion option.
   C. Cap on a floating-rate security.

8. The price of a 10-year zero coupon bond with a current YTM of 9.4% is 39.91. If the YTM increases to 9.9%, the price will decrease to 38.05. If the YTM decreases to 8.9%, the price will increase to 41.86. The effective duration is *closest* to:
   A. 9.38.
   B. 9.48.
   C. 9.55.

9. The effects of a decrease in interest rate (yield) volatility on the market yield of a debt security with a prepayment option and on a debt security with a put option are *most likely* a(n):

	Prepayment option	Put option
A.	Increase	Decrease
B.	Decrease	Increase
C.	Decrease	Decrease

10. Bond A has an embedded option, a nominal yield spread to Treasuries of 1.6%, a zero-volatility spread of 1.4%, and an option-adjusted spread of 1.2%. Bond B is identical to Bond A except that it does not have the embedded option, has a nominal yield spread to Treasuries of 1.4%, a zero-volatility spread of 1.3%, and an option-adjusted spread of 1.3%. The *most likely* option embedded in Bond A, and the bond that is the better value, are:

	Embedded option	Better value
A.	Put	Bond A
B.	Call	Bond A
C.	Call	Bond B

11. A bank loan department is trying to determine the correct rate for a 2-year loan to be made two years from now. If current implied Treasury effective annual spot rates are: 1-year = 2%, 2-year = 3%, 3-year = 3.5%, 4-year = 4.5%, the base (risk-free) forward rate for the loan before adding a risk premium is *closest* to:
   A. 4.5%.
   B. 6.0%.
   C. 9.0%.

12. Compared to mortgage passthrough securities, CMOs created from them *most likely* have:
   A. less prepayment risk.
   B. greater average yields.
   C. a different claim to the mortgage cash flows.

13. The arbitrage-free approach to bond valuation *most likely*:
   A. can only be applied to Treasury securities.
   B. requires each cash flow to be discounted at a rate specific to its time period.
   C. shows that discounting each cash flow at the yield to maturity must result in the correct value for a bond.

14. Which of the following statements *least accurately* describes a form of risk associated with investing in fixed income securities?
    A. Credit risk has only two components, default risk and downgrade risk.
    B. Other things equal, a bond is more valuable to an investor when it has less liquidity risk.
    C. Bonds that are callable, prepayable, or amortizing have more reinvestment risk than otherwise equivalent bonds without these features.

Self-Test Answers: Fixed Income Investments

1. **A** Duration is a linear measure, but the relationship between bond price and yield for an option-free bond is convex. For a given decrease in yield, the estimated price increase using duration alone will be smaller than the actual price increase.

2. **A** First, determine the semiannual yield to maturity:

   N=38, PMT=3.5, PV= –91.16, FV=100 CPT→I/Y = 3.9534.

   Second, retain TVM info and enter:

   N = 34 CPT→PV = 91.60 for a gain of (91.60 – 91.16)/91.16 = 0.4827%.

3. **C** The market segmentation theory is based on the idea that different market participants (both borrowers and lenders) have strong preferences for different segments of the yield curve. If expectations are that future short-term interest rates will be falling enough, then the yield curve could be downward sloping even given that there is an increasing premium for lack of liquidity at longer maturities. A humped yield curve is consistent with expectations that short-term rates will rise over the near term and then decline.

4. **A** External credit enhancements are financial guarantees from third parties that generally support the performance of the bond. Portfolio insurance is not a third party guarantee.

5. **A** Effective duration must be used with bonds that have embedded options.

   $\Delta P = (-)(ED)(\Delta y) + (C)(\Delta y)^2$

   $\Delta P = (-)(6)(0.0025) + (62.5)(0.0025)^2 = -0.015 + 0.00039 = -0.014610\%$ or -1.461%

6. **B** The bond with the highest duration will benefit the most from a decrease in rates. The lower the coupon, lower the yield to maturity, and longer the time to maturity, the higher will be the duration.

7. **B** A conversion provision is an embedded option that favors the buyer, not the issuer, so buyers will accept a lower YTM with a conversion option. Call options and caps favor the issuer and increase the YTM that buyers will require.

8. **C** Effective duration = (41.86 – 38.05)/(39.91 × 2 × 0.005) = 9.546. Note that the price changes are based on a 50 basis points change in yield, so $\Delta y = 0.005$.

9. **B** A decrease in yield volatility will decrease the values of embedded options. The security holder is short the prepayment option. The decrease in the value of the prepayment option increases the value of the security, and the required yield will decrease. The security holder is long the put option so the value of a putable bond will decrease with a decrease in yield volatility and the required yield will increase.

10. **C** Since the OAS is less than the Z-spread for Bond A, the effect of the embedded option is to decrease the required yield, so it must be a call option and not a put option. The OAS is the spread after taking out the effect of the embedded option. Since the OAS is higher for Bond B, it represents the better value after adjusting for the value of the call in Bond A.

11. **B** The forward rate is $[1.045^4/1.03^2]^{1/2} - 1 = 6.02\%$, or use the approximation $[4.5(4) - 3(2)]/2 = 6$.

12. **C** CMOs are created to have different claims to the cash flows (principal, scheduled repayments, prepayments) than those of the underlying mortgage passthrough securities. On average, the yield will likely be lower on the CMO, since the reason to create them is to lower overall funding costs. They can have more or less prepayment risk, but on average will have the same prepayment risk as the underlying MBS.

13. **B** The arbitrage-free valuation approach discounts each cash flow at a discount rate specific to its maturity. For Treasury securities these discount rates are theoretical Treasury spot rates. For non-Treasury securities, these discount rates are Treasury spot rates plus a spread to account for liquidity risk, credit risk, and any other relevant risks that differ from those of a Treasury bond of similar maturity.

14. **A** Even if a bond does not default and is not downgraded, it still faces credit spread risk as the premium in the market for the bond's credit risk may increase. Lower liquidity risk (i.e., higher liquidity) is preferred by investors, reduces a bond's required rate of return, and increases its value, other things equal. Reinvestment risk is higher for callable, prepayable, or amortizing bonds as all these features lead to a greater probability of receiving principal repayment earlier, which means there are more funds to be reinvested over the life of the bond.